COMPARISON OF ULTIMATE LOAD CAPACITIES OF REINFORCED CONCRETE BEAMS BASED ON TYPICAL DESIGN SPECIFICATIONS

by

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The development of constructions raised increased demands for the design accuracy of r.c. beams in combined axial and torsion stresses. The first upswing in the '60s (COWAN, LEONHARD, THIENKOV, GESUND, GVOSDEV) mainly concerned pure torsion in the elastic range.

The second surge by the early '70s involved extended tests (LAMPERT, SZILÁRD, HSU ZIA, THÜRLIMANN, COLLINS, LESSIG) of deformometry on reinforced and prestressed beams and plates under static and dynamic axial stresses combined with torsion in each three stress states. Results of the numerous failure theories and approximate calculation methods are involved in most national standard specifications and international recommendations in virtue.

It is still a question whether ultimate load capacities under combined stresses calculated according to different national standards for the same beam exhibit typical differences or designing the beam for the same stresses according to any standard leads to the same structure?

Since the beam subject to bending moments and torque undergoes greater deformations than under shear and normal forces, these latter may be omitted by adequately selecting stresses and reinforcement.

Let us consider ultimate load capacities of a r.c. beam of rectangular cross section calculated according to different recommendations and standard specifications.

Let the cross section have strong lower reinforcement and adequate stirrups. Bending moment will cause tension in the bottom fibre and the effect of upper longitudinal reinforcement on bending will be negligible. Other cross sectional dimensions and reinforcement percentages will be assumed according to the Annex, in conformity with the practice.

Examinations will involve:	Hungarian Standard MSz 15022/1—71	[1]
	CEB Recommendations (71)	[2]
	ACI Standard 318—71 and	[3]
	СНиП II.—21—75	[4].

Here CEB Recommendations represent West-European approach, ACI Standard the overseas one, while СНиП, fundamentally different, may be of interest especially in case of revising the Hungarian standard, since it is essentially the basis of COMECON recommendations.

Design principles in different specifications

In a bar of homogeneous cross section, torsion causes shear stresses increasing linearly with the distance from the torsion center. Contribution of the central core of the cracked cross section to the load capacity is negligible. Thus, the r.c. beam assumes the torsion in the close vicinity of the cross section circumference, where the tensile principal stresses of the shell in shear are borne either by the helical stirrups or by the longitudinal and transversal reinforcement in common, while the compressive principal stresses are balanced by the concrete compressive strength (Fig. 1).

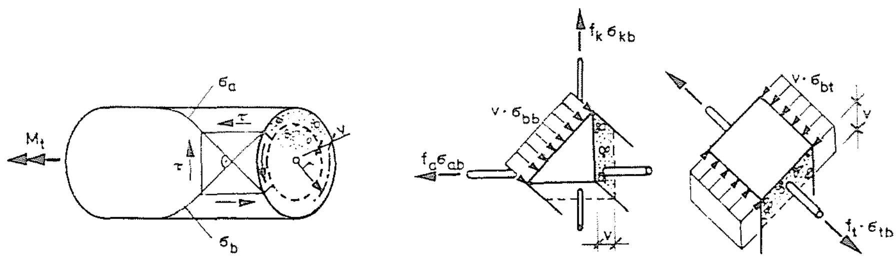

Fig. 1

Stresses τ in the resulting thin-walled cross section of nominal thickness v may be computed by the BREDT formula. Reinforcement in the shear shell is advisably such as to provide equivalent action longitudinally and transversally.

In case of combined stresses, normal stresses due to bending are superposed, hence flexural steels may be strengthened by torsion reinforcement in the vicinity of the tensile fibre, while the rest may be distributed uniformly along the cross section circumference.

Adequate design specifications provide for the oblique principal stresses in compression due to combined effects nowhere to exceed the ultimate values in the concrete shell in shear. Hence, the combined effect has been decomposed into components and the quantities of longitudinal and transversal reinforcement calculated separately for bending and for torsion.

This is the essential in specifications [1], [2] and [3], the latter two being concerned with effective stresses τ and indicating solid cross section equivalents of nominal wall thickness v, specifying the range of validity of the design method as a function of the cross sectional form. The intermediate step of calculating stresses τ is omitted in [1], and lower than ultimate concrete stresses are guaranteed by construction rules though less strict than those in [2] and [3]. Combined stresses τ are limited by [2], taking most adverse cases into consideration, but summing of reinforcement areas according to [1] and [3], or application of another design method if justified by tests is admitted. On the basis of tests, [4] distinguishes three failure modes, where the skew concrete compressive zone develops in the compressed, lateral or tensile side, depending on the ratio of torsion to bending moments. Specific ultimate stress diagrams of the tested beam are seen in Fig. 2.

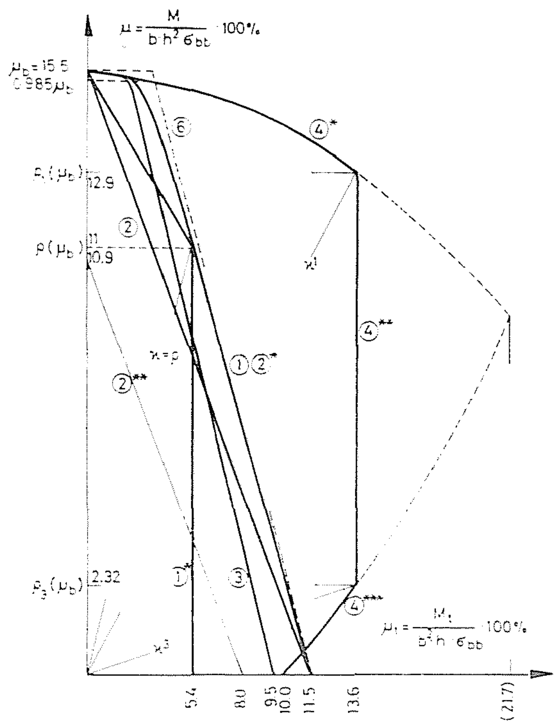

Fig. 2

Calculation methods

1. Hungarian Standard MSz 15022/1—71 specifies conditions

$$T_H = 0.25\ b \cdot h\sigma_{bH} - bh\frac{M_t}{K_t} \le T \tag{1}$$

to be met by the concrete, and

$$M_{tH} = 2\sigma_{bH} \cdot \frac{F_k}{t} \cdot F_t \le M_t \tag{2}$$

or

$$M_{tH} = 2\sigma_{aH}\frac{F_a}{K} F_t \le M_t \tag{3}$$

by the steel, and specifies at the same time to consider torsional reinforcement as reinforcement excess [curve ①] and to distribute longitudinal reinforcement in torsion uniformly along the cross section circumference. Curve ①*. considers uniformly distributed longitudinal reinforcement alone, with the deduction of bars in bending.

No concrete and steel grade are limited. Closed stirrups are specified in any standard, hence also here.

2. CEB Recommendations calculate torsional shear stresses in equilibrium according to

$$\tau_t = \frac{M_t}{2F_t \cdot v} \tag{4}$$

v being the theoretical wall thickness, in the actual case the lower from $b/6$ and $b_k/5$ (Fig. 3).

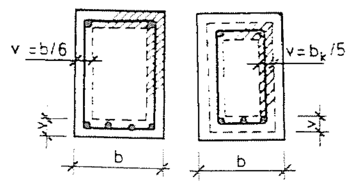

Fig. 3

Conditions for web stresses and torsional stresses are

$$\frac{\tau_0}{\tau_0^{\max}} + \frac{\tau_t}{\tau_t^{\max}} \leq 1 \tag{5}$$

[curve ②], and for bars in pure torsion

$$\frac{F_a}{K} = \frac{\Sigma F_k}{t} = \frac{M_t}{2F_t\,\sigma_{aH}} \tag{6}$$

essentially the same as in [2] and [3]. (Here $\sigma_{aH} = \sigma_{kH}$, and $\tau_0^{\max}$ is the tangential web stress due to pure bending in ultimate shear.) Summing of longitudinal bars in bending and in torsion is allowed in [1] [curve ②*]. In case of limited deformation, no 1 but only 0.7 is admitted in Eq. (5) [curve ②**]. CEB stipulates concrete and steel grades.

3. The ACI Standard specifies stirrups for shear and torsion of the selected cross section type to meet the minimum condition

$$\frac{F_k^{(\tau)} + 2F_k^{(\tau_t)}}{t} \cdot \sigma_{kH} \geq \overset{(3.52)}{50} \cdot b \tag{7}$$

where

$$\tau_t^{\max} < 1.5\,\sqrt{\sigma_{bH}}\ . \tag{8}$$

Calculating here the shear stresses by

$$\tau = \frac{T}{\Phi b \cdot h} \tag{9}$$

and torsion stresses by

$$\tau_t = \frac{3M_t}{\Phi b^2 h} \tag{10}$$

the concrete is required not to take more than:

$$\tau_t^{\text{concrete}} = \frac{\overset{(0.636)}{2.4}\sqrt{\sigma_{bH}}}{\sqrt{1 + \left(1.2\,\dfrac{\tau}{\tau_t}\right)^2}} \tag{11}$$

or, the total nominal torsion stress not to exceed:

$$\tau_t^{\text{total}} = \frac{\overset{(3.18)}{12}\sqrt{\sigma_{bH}}}{\sqrt{1 + \left(1.2\dfrac{\tau}{\tau_t}\right)^2}} \tag{12}$$

Φ in (9) and (10) depends on the stress type, in the actual case $\Phi = 0.85$.

Here too, the torsion reinforcement has to be considered as excess of a value:

$$F_k = \frac{(\tau^{\max} - \tau_t^{\max})\, b^2 h t}{3\alpha_t \cdot b_k \cdot h_k\, \sigma_{kH}} \tag{13}$$

where:

$$\alpha_t = 0.66 + 0.33\,\frac{h_k}{b_k} \leq 1.50 \tag{14}$$

and

$$t \leq \frac{b_k + h_k}{4} \leq \overset{(30)}{12''} \,. \tag{15}$$

Under torsion, the ratio of longitudinal to transversal excess reinforcement will be the least from

$$F_a = 2\,\frac{F_k}{t}\,(b_k + h_k) \tag{16}$$

$$F_a = \left(\frac{\overset{(28.1)}{400}\, b \cdot t}{\sigma_{bH}} \cdot \frac{\tau_t^{\max}}{\tau^{\max} + \tau^{\max}} - 2F_k\right)\frac{F_k}{2t} \tag{17}$$

$$F_a = \left(\frac{\overset{(28.1)}{400}\, b \cdot t}{\sigma_{bH}} \cdot \frac{\tau_t^{\max}}{\tau^{\max} + \tau_t^{\max}} - \frac{\overset{(3.52)}{50}\, bt}{\sigma_{kH}}\right)\frac{F_k}{2t} \,. \tag{18}$$

There are further stipulations on the theoretical wall thickness, quantity and quality distribution of the reinforcement and on the concrete grade. (Constants in parentheses in the formulae refer to the metric system, and without, to Anglo-Saxon units.)

4. СНиП differs from the former ones by distinguishing between three modes of failure, for a maximum torque

$$M_t = 0.1\ \sigma_{bH}\ b^2 h\ . \tag{19}$$

In the first mode of failure (Fig. 4), mainly under high bending and low torsion moment, a skew hinge in compression develops at the compressive side corresponding to bending, at an angle to cause the least deformation work to internal forces. Depth x_1 may be calculated from bending.

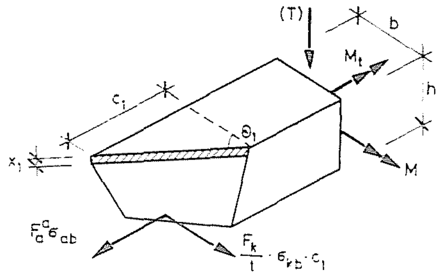

Fig. 4

In the second mode of failure, especially under high shear forces and low bending moments, plastic hinge develops laterally, as seen in Fig. 5.

The third mode of failure is produced by a very high torque and low bending moment and low shear force, according to Fig. 6.

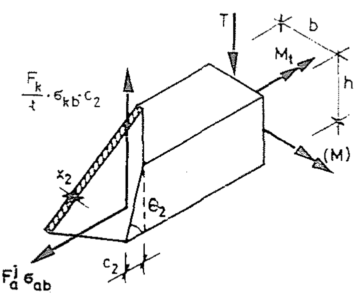

Fig. 5

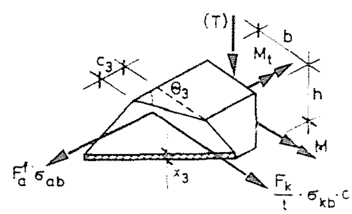

Fig. 6

In any mode of failure, the ultimate torque vs. bending moment:

$$M_t = F_a^i\, \sigma_{aH}(h^i - 0.5x^i)\,\frac{1 + \gamma\delta\beta^2}{k\beta + \varkappa} \tag{20}$$

where

$$\gamma = \frac{F_k\,\sigma_{kH}\cdot b}{F_a^i} \le \frac{0.5}{1 + 2\varkappa\sqrt{\delta}} = \gamma_{\min} \tag{21}$$

$$\ge \frac{1.5}{1 + 2\varkappa\sqrt{\delta}} = \gamma_{\max}\ .$$

(for $\gamma < \gamma_{\min}$ the $F_a^i\,\sigma_{aH}$ values have to be multiplied by $\frac{\gamma}{\gamma_{\min}}$ and i refers to the side corresponding to the mode of failure) hence:

$$\delta = \frac{b}{2h+b} \tag{22}$$

and

$$\beta = \frac{c}{b} = \tan\theta \tag{23}$$

the ratio of torsion to bending moment being:

$$\varkappa = \frac{M}{M_t}. \tag{24}$$

For $\varkappa = \frac{M}{M_t}, k = 1$ in schemes 1 and 2, resp., a case only possible for $M_t \leq 0.5T \cdot h$ and for $\varkappa = 0$, $k = 1 + \frac{T \cdot h}{2M_t}$. According to the third scheme, for $\varkappa = -\frac{M}{M_t}$, $k = 1$, c cannot exceed $(2h + b)$.

According to LESSIG [8], in conformity with the minimum condition of deformation work, for $\frac{dM_t}{d\theta} = 0$, in the first case

$$\tan\theta = -\varkappa + \sqrt{\varkappa^2 + \frac{(1+2\alpha)}{\gamma}} \tag{25}$$

($\alpha = \frac{h}{b}$ being the cross section slenderness), while introducing for the third case the ratio of lower to upper reinforcement:

$$R = \frac{F_a^f\,\sigma_{aH}^f}{F_a^a\,\sigma_{aH}^a} \tag{26}$$

$$\tan\theta = \varkappa + \sqrt{\varkappa^2 + \frac{(1+2\alpha)}{\gamma}}\quad . \tag{27}$$

Hence in the first and the third case, ultimate torque vs. ultimate bending moment:

$$M_{t_1} = M_H \frac{2\gamma}{1+2\alpha}\left(\sqrt{\varkappa^2 + \frac{1+2\alpha}{\gamma}} - \varkappa\right) \tag{28}$$

[see curve ④ *] or:

$$M_{t_2} = M_H \frac{2\gamma}{R(1+2\alpha)}\left(\sqrt{\varkappa^2 + \frac{(1+2\alpha)R}{\gamma}} + \varkappa\right) \tag{29}$$

[see curve ④ ***].

In the second case, for a ratio of shear to torsion:

$$\delta^* = \frac{T \cdot b}{2M_t} \tag{30}$$

or slenderness of the "active cross section":

$$\alpha^* = \frac{h_k}{b_k} \tag{31}$$

then, similarly as before:

$$M_{t_2} = M_H \frac{1}{1+\delta^*} \cdot \frac{\alpha}{\alpha^*} \sqrt{\frac{2(1+R)\gamma}{2\alpha}} \tag{32}$$

[see curve ④**].

Similarly to the former, [4] stipulates lowest concrete and highest steel grade and reinforcement composition design methods.

Evaluation of load capacity ranges

In all procedures referred to, the ultimate bending moment has an analogous definition. Hence for $\mu_t = 0$, all curves join a common point μ_H in Fig. 2.

Different specifications containing different values for the ultimate torque, ultimate stresses are advisably referred to ultimate pure bending or torsion stresses rather than to ultimate load capacity curves (Fig. 7).

Curve ⑤ in Fig. 7 represents a relationship suggested by ERSOY and FERGUSON [10] and supported by several tests in crackless and cracked ultimate elastic condition. It is interesting to see curves ①, ②* and ③ to but slightly dif-

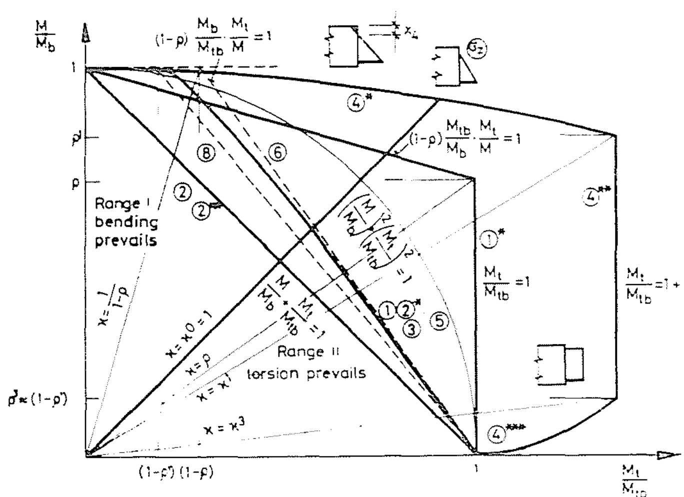

Fig. 7

fer up to $\frac{M}{M_H} = 0.98$ from straight line ⑥ tangential to the previous one at $\frac{M}{M_H} = 0$. In the range ϱ being the $\frac{M}{M_H}$ value where — deducing bars needed to take bending stresses — the remaining longitudinal bars are uniformly distributed along the cross section circumference (inflexion of curve ①*), in the range $\frac{M_t}{M_{tH}} \geq 1 - \varrho$ ⑥ is obtained from

$$(1-\varrho)\frac{M_H}{M_{tH}} \cdot \frac{M_t}{M} = 1 \ . \tag{33}$$

ϱ depends on the ratio of lower to upper reinforcement according to (26):

$$\varrho = \varrho^\circ R \ . \tag{34}$$

Although replacement of curves ①, ②* and ③ by line ⑥ is an approximation detrimental to safety, it is still safer than curve ④ better describing real modes of failure and adequately supported by formulae, simplifying calculations.

(A further simplification possibility is to apply a line corresponding to curve ⑤, writing ϱ for ϱ' in (3). Curve ④ hints to keep the following peculiarities of the behaviour of r.c. beams in combined axial load and torsion in mind:

a) for $\varkappa < \varkappa_3$, the ultimate load capacity in torsion is slightly (ϱ^*) increased by flexural compression (also true for external or prestressing normal force), but

b) in the range $\varkappa < \varkappa_1$, torsion capacity abruptly decreases.

On cross section sides likely of developing in fact the skew compressed zone, and on the adjacent sides, "torsion excess bars" do not add much to the ultimate load capacity in torsion. It is advisable therefore to arrange the longitudinal torsional steel calculated according to [1], [2], [3] on the side opposite to the concrete compressive zone developing according to [4] rather than to uniformly distribute it.

Ultimate load capacity in combined shear and torsion is beneficially affected by closed stirrups with a double leg on the beam side exposed to cracking due to shear and torsion of the same direction.

Legend of symbols not defined in the text

M, M_t = moment and torque
T = shear
σ_a, σ_k, σ_b = longitudinal, transversal steel and concrete stress
τ, τ_0, τ_t = tangential stress due to shear, moment and torque
F_a, F_k, F_t = longitudinal, transversal and helical bar or stirrup area
K = length of stirrup
b, h = width and depth of the concrete cross section
b_k, h_k = horizontal and vertical stirrup length
t = stirrup spacing
x = depth of the compressed zone
c = projection of the compressed zone midline on the beam axis

Subscripts:

i = mode 1, 2, 3 of failure
H = ultimate force or stress.

Superscripts:

a = bottom
f = top.

Summary

Typical design specifications are involved to compare ultimate load capacities in combined bending and torsion of a common type of cross section.

An approximate design method, simpler to apply than the Hungarian one, and an expedient arrangement of torsion bars based on the typical behaviour of beams under combined stresses are suggested.

References

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Annex

The selected cross section is a rectangular one, of a ratio $\alpha = 2$, its closed stirrups are symmetrical in the cross section plane, giving $\alpha^* = 2.27$. The ratio of longitudinal to transversal reinforcement $\frac{F_k}{t} = 17.1 \frac{F_a}{K}$, the ratio of left to right side stirrups is 0.333, the ratio of lower to upper reinforcement is 3.43 and of the left to right side one 1.0. the ratio of concrete to steel stress is 21. of longitudinal bar to stirrup stress is 1.235.

With these proportions, for $\varrho = \frac{1}{\sqrt{2}}$ and $\varrho_0 = 2.44$, $\varrho^* = 0.368$ and $\varrho' = 0.851$, $\varkappa_1 = \frac{3}{5}$, $\varkappa_3 = \frac{1}{9}$.

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* In Hungarian

** In Russian